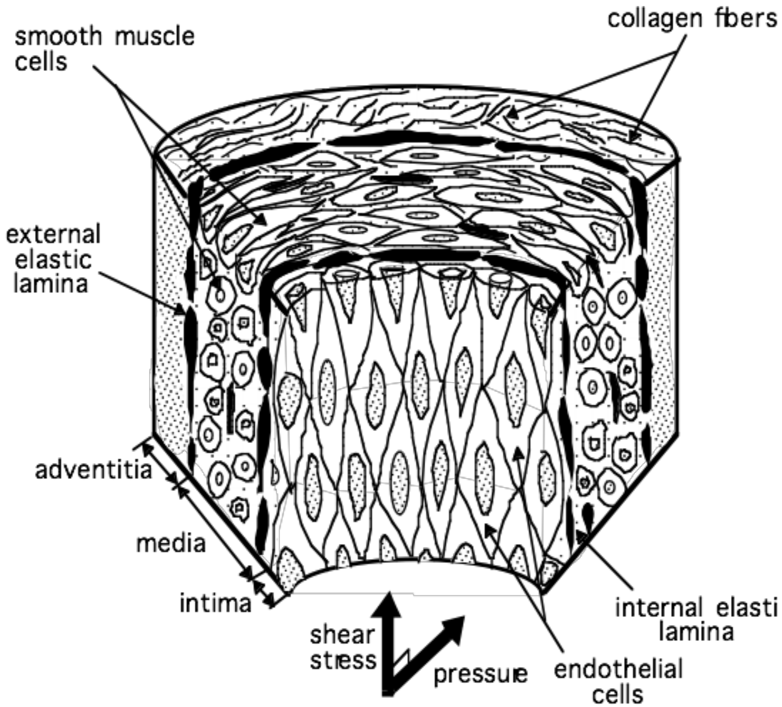
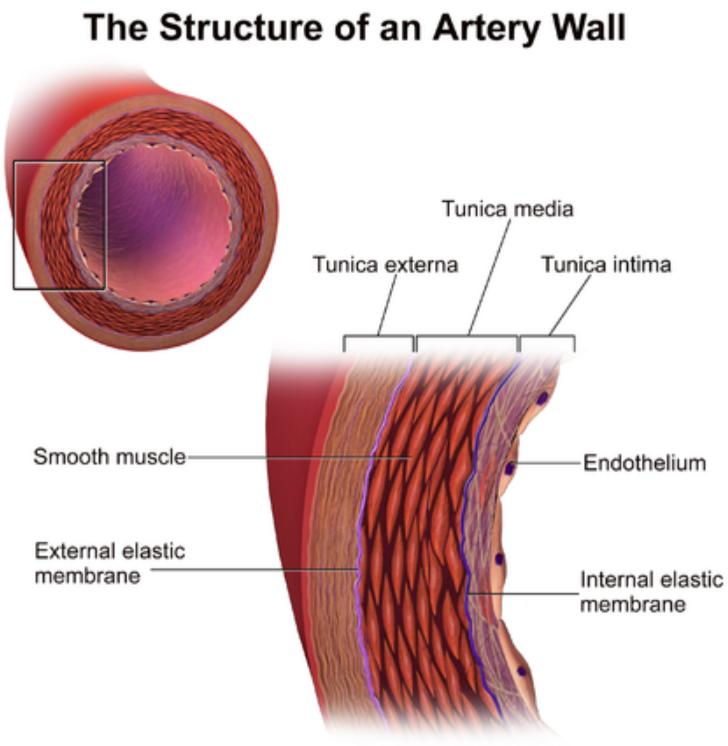
Arterial wall mechanics

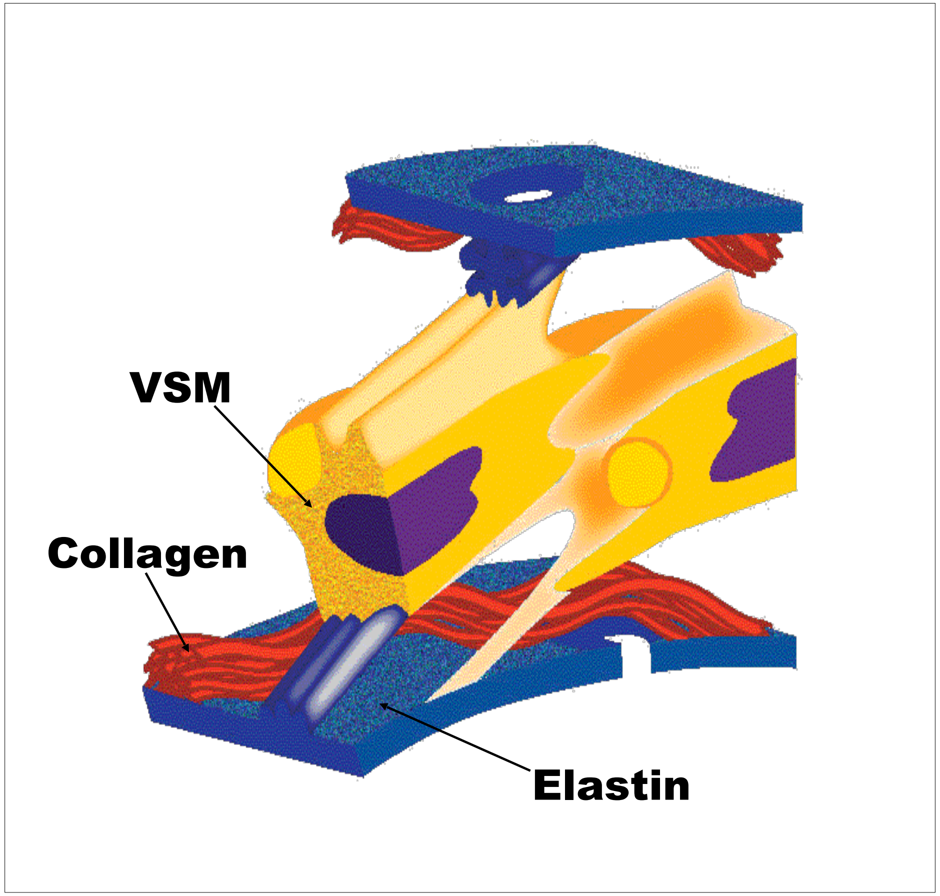
- Arterial wall structure and composition
- Elastic properties
  - Pressure-diameter relation
  - Incompressibility
  - Compliance
  - Elastic modulus
  - Viscoelasticity
- Wall stress, law of Laplace
- Prestress
- Active properties
  - Smooth muscle cell contraction
  - Myogenic response
  - Vasomotion

Structure and composition of the arterial wall





Structure and composition of the arterial wall

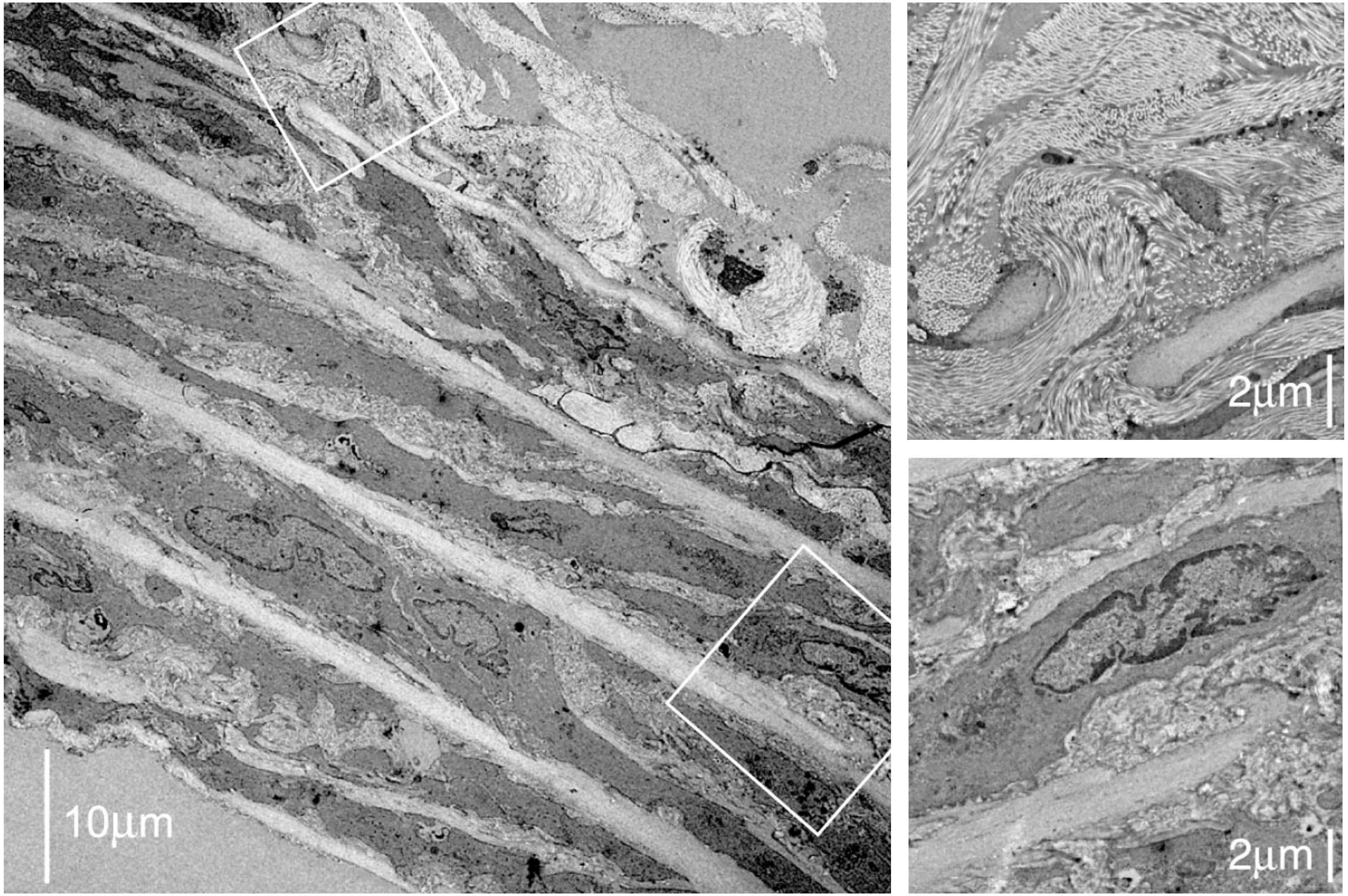


Elastin
- large deformations
- elastic modulus E_{el}= 600 kN/m^2

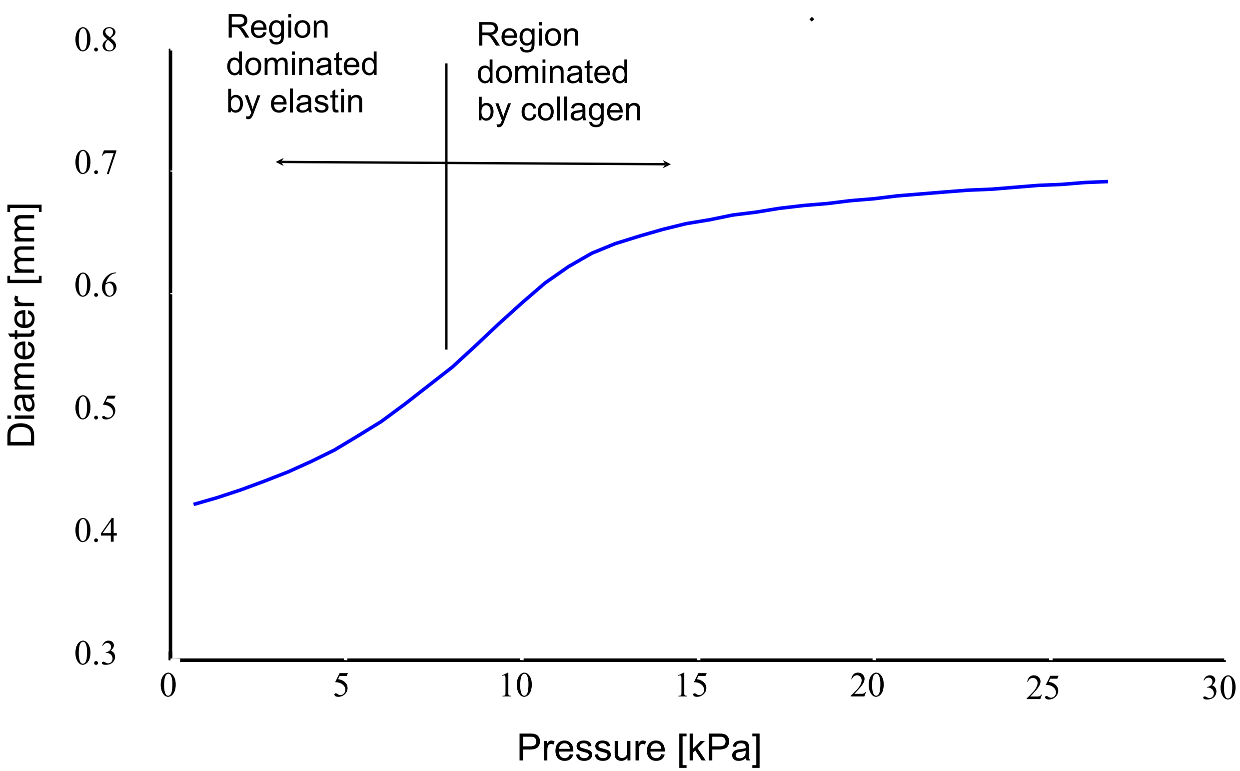
Collagen
- wavy fibers
- elastic modulus E_{col}= 1000 x E_{el}

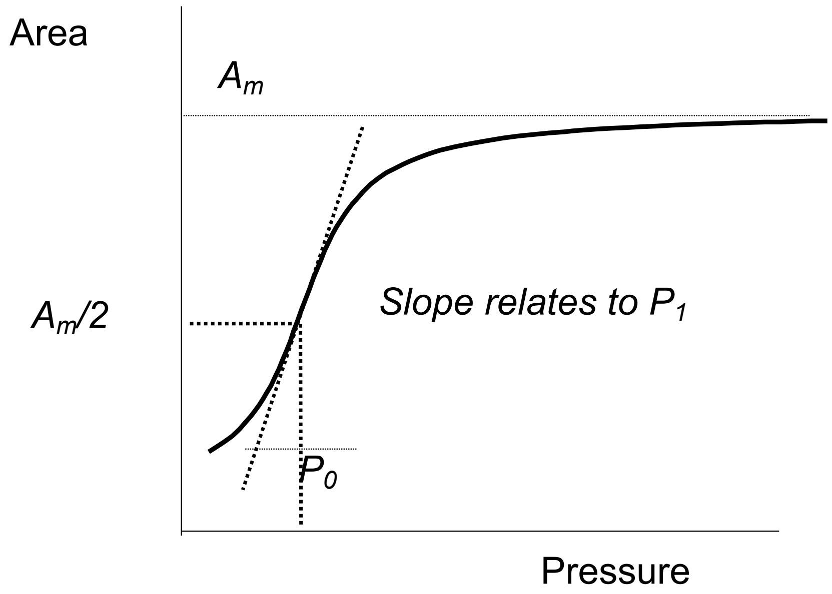
Smooth muscle cells
- contraction/relaxation
- negligible elasticity in passive state

Histology of the arterial wall



Pressure-diameter relation

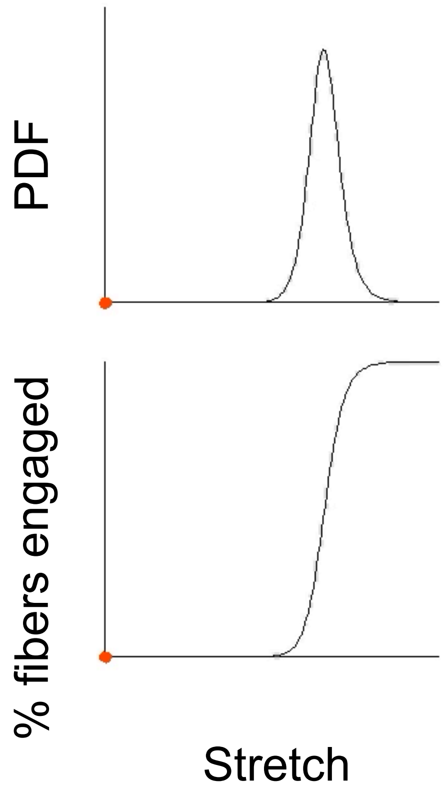


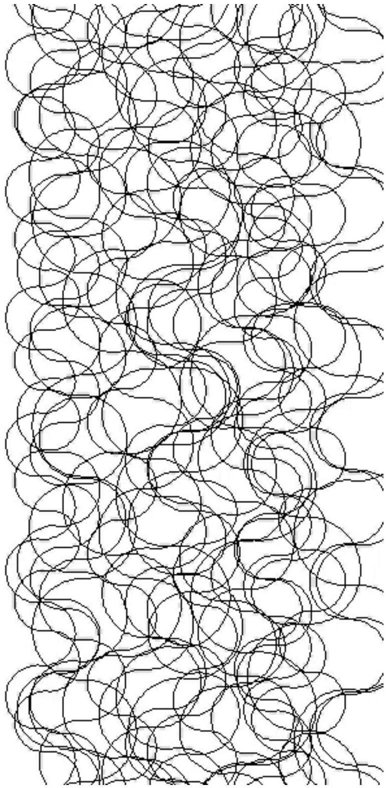


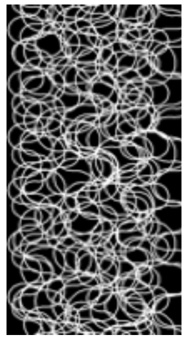
$$A = A_m\left[\frac{1}{2} + \tan^{-1}\left(\frac{P - P_o}{P_1}\right)\right]$$

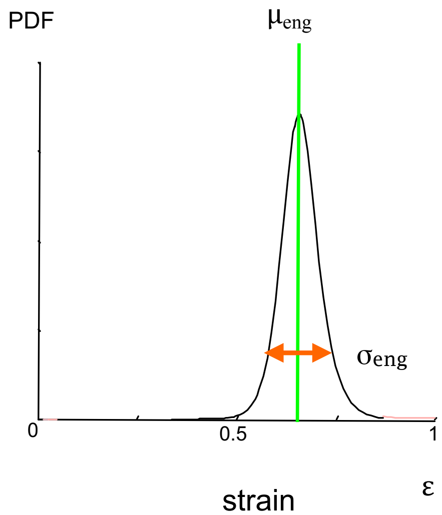
Langewouters et al., 1984

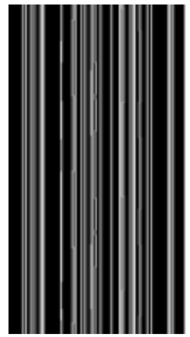
Role of collagen in the pressure-diameter relation





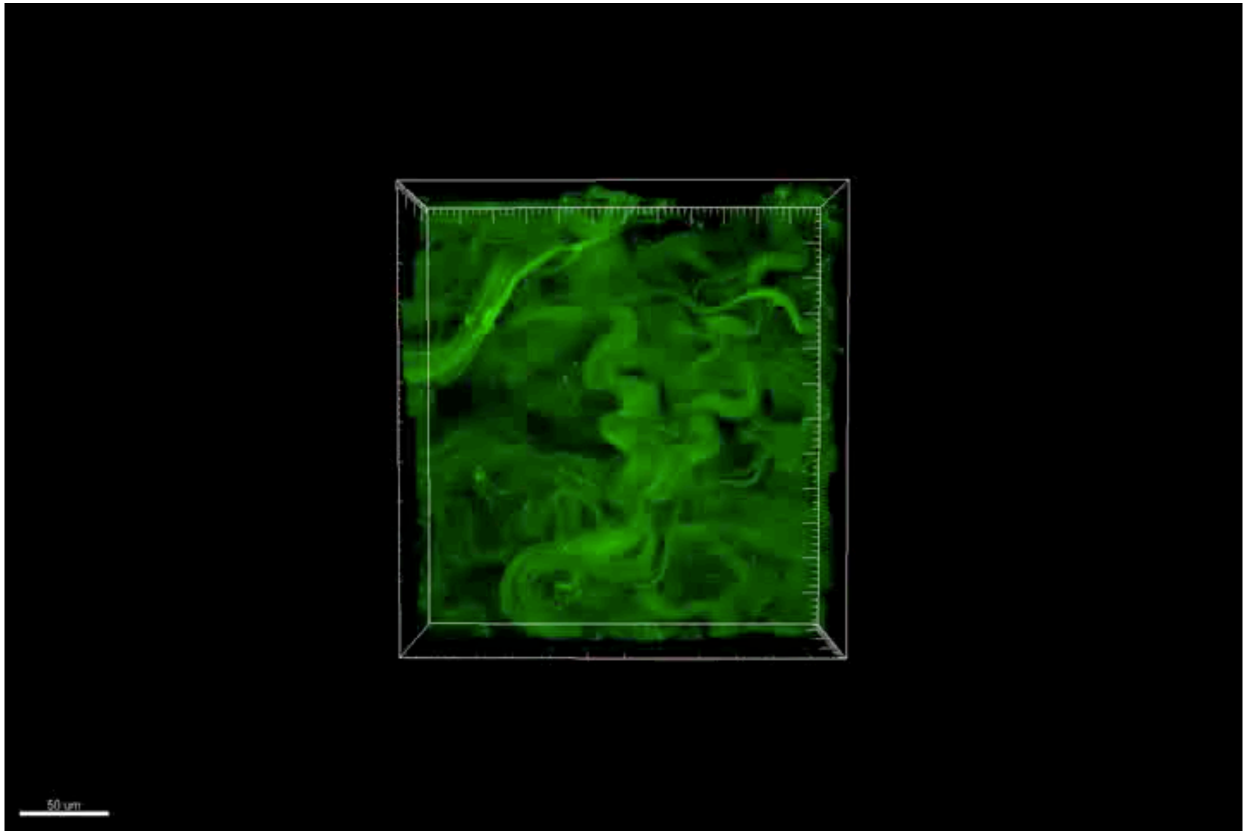






- Statistical description of collagen engagement
- Mean engagement strain (μ_{eng}) and width of PDF (σ_{eng}) are two key parameters influencing collagen engagement and arterial mechanics
- Analysis shows that less than 10% of collagen fibres are engaged in a physiological artery

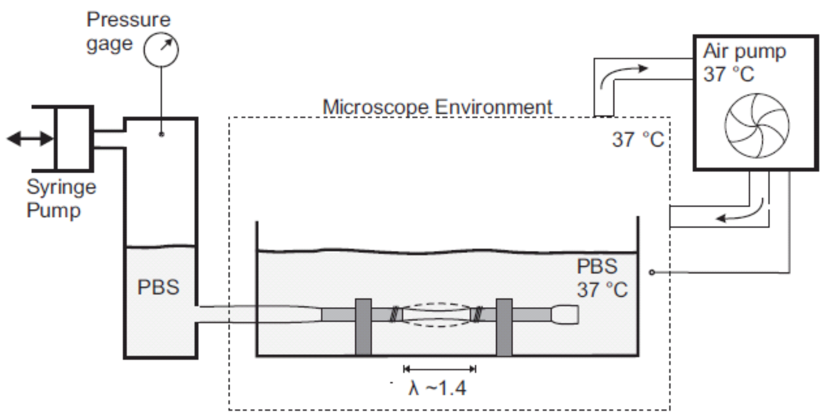
Visualization of collagen engagement under inflation



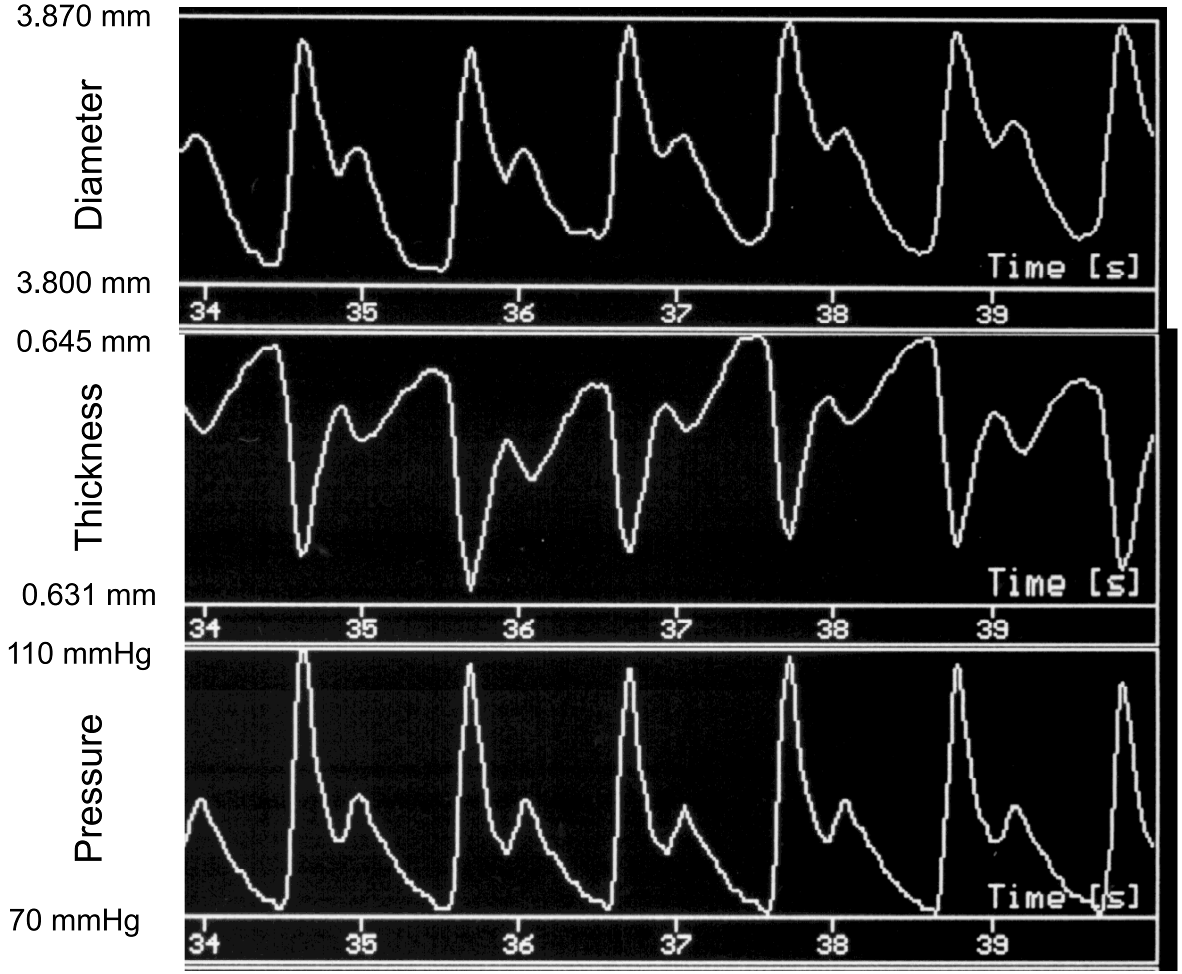
Experimental set-up:

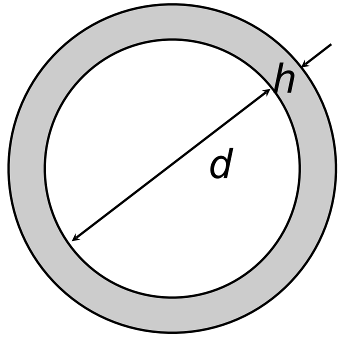
confocal microscope

CNA35-OG488 fluorescence marker



Incompressibility





Incompressible wall:

$$\frac{\pi}{4}(d+2h)^2 - \frac{\pi}{4}d^2 = ct$$

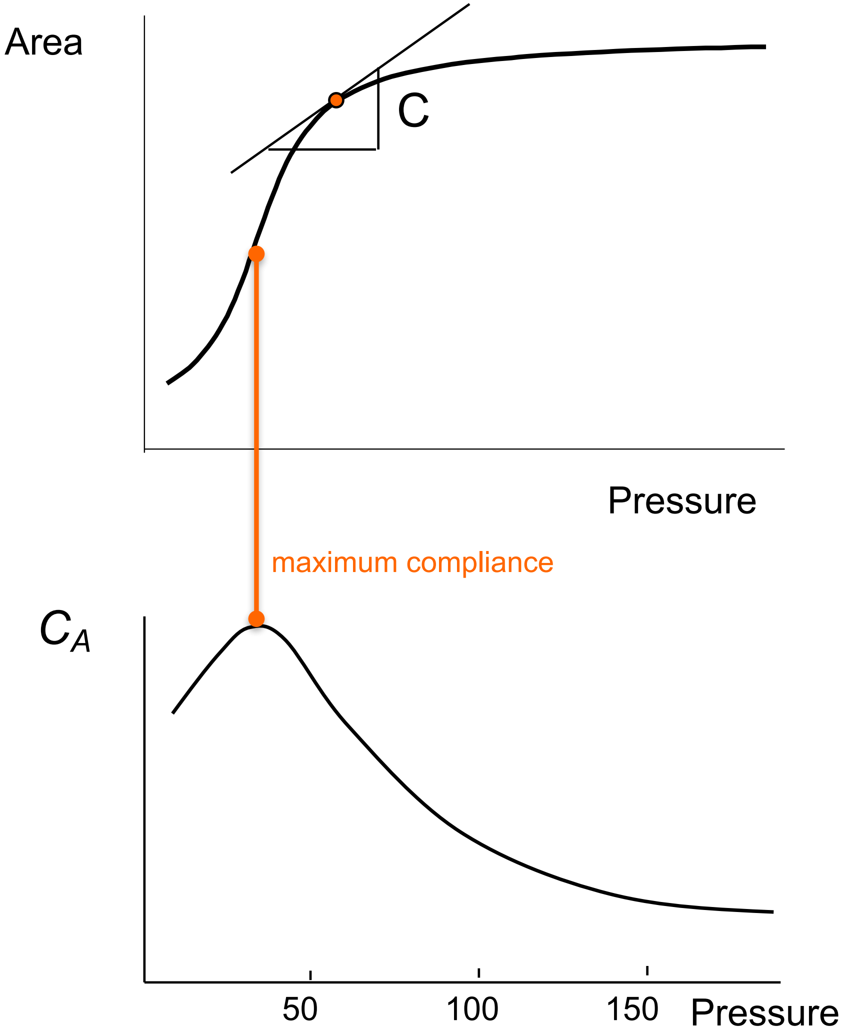
$$\Rightarrow d^2 + 4dh + 4h^2 - d^2 = ct$$

$$\Rightarrow dh + h^2 = ct$$

For thin incompressible wall (h << d):

$$dh = ct$$

Compliance & Distensibility



Compliance: $$C_A = \frac{\partial A}{\partial P}$$ Area compliance

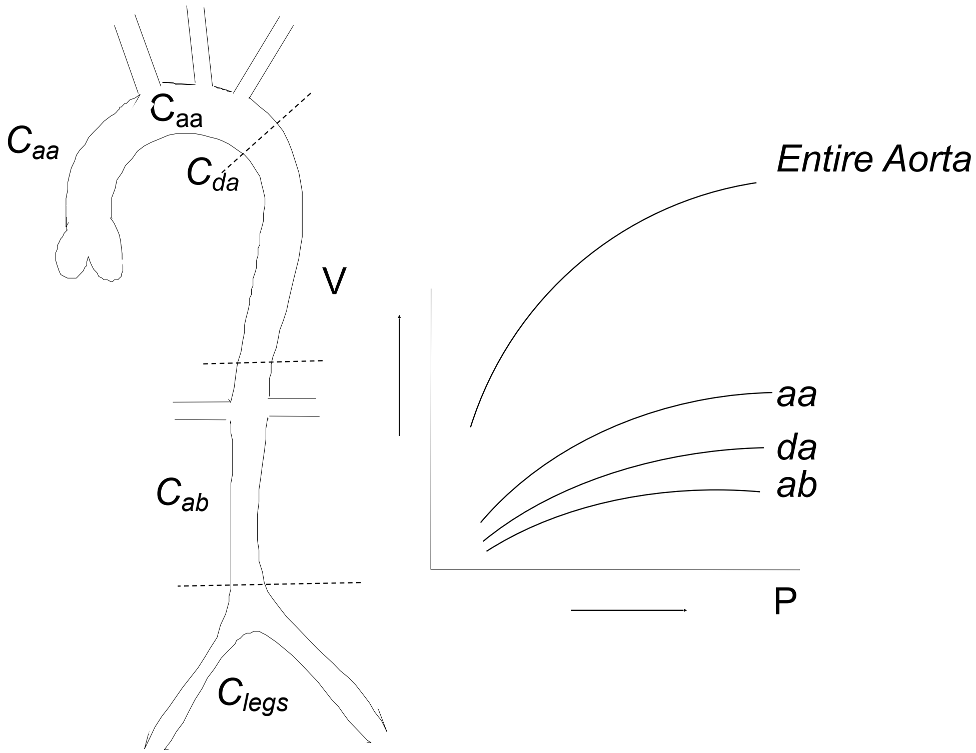
$$C_V = \frac{\partial V}{\partial P}$$ Volume compliance

Distensibility: $$D = \frac{1}{A}\frac{\partial A}{\partial P}$$

$$D = \frac{1}{V}\frac{\partial V}{\partial P}$$

- Compliance is highly dependent on pressure
- For physiological pressures (60 < P < 140 mmHg), compliance decreases when pressure increases

Compliance of the arterial system: addition of compliances



$$C_{aa} + C_{da} + C_{ab} =$$

$$\frac{\Delta V_{aa}}{\Delta P} + \frac{\Delta V_{da}}{\Delta P} + \frac{\Delta V_{ab}}{\Delta P} =$$

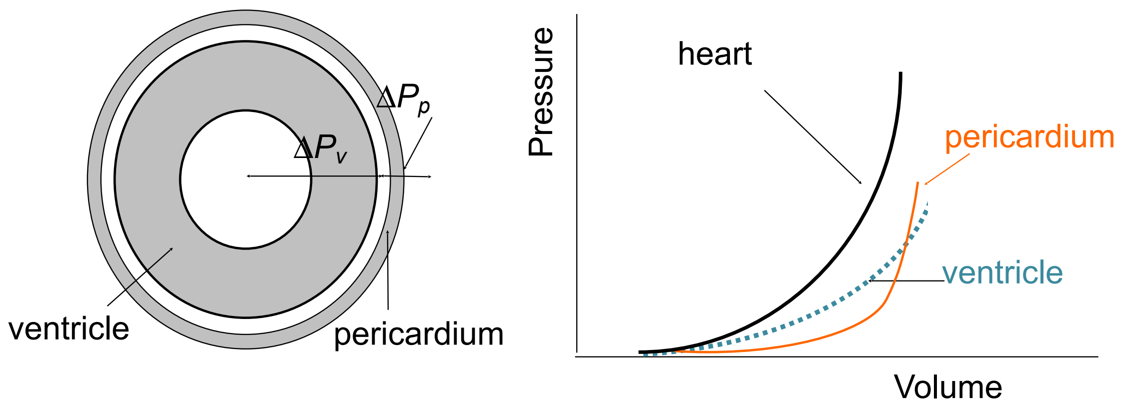
$$\frac{\Delta V_{aa} + \Delta V_{da} + \Delta V_{ab}}{\Delta P} =$$

$$\frac{\Delta V_{entire\,aorta}}{\Delta P} = C_{entire\,aorta}$$

<u>Conclusion</u>: The compliances of the different sections can be added to obtain total aortic compliance.

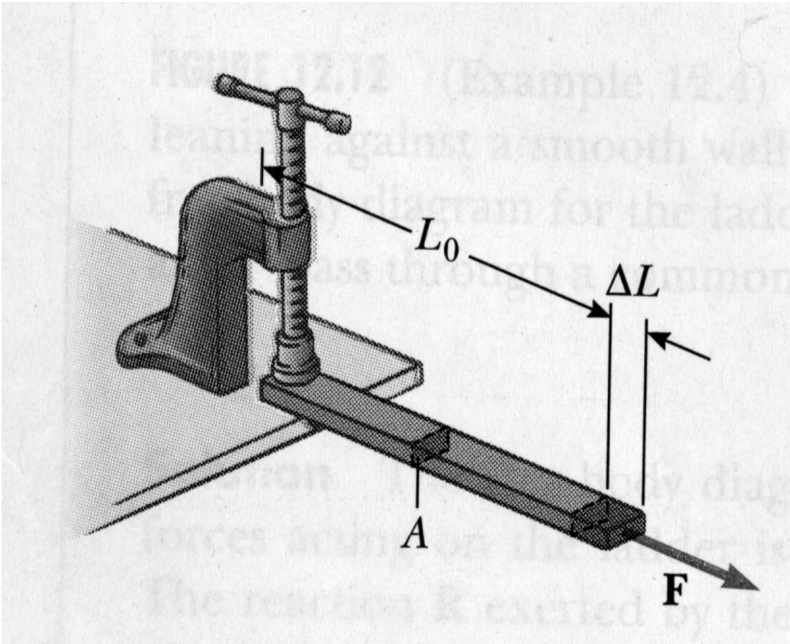
Elastance

The elastance E is the inverse of compliance C_v : $$E = \frac{dP}{dV} = \frac{1}{C_v}$$



$$E = \frac{\Delta P}{\Delta V} = \frac{\Delta P_v + \Delta P_p}{\Delta V} = \frac{\Delta P_v}{\Delta V} + \frac{\Delta P_p}{\Delta V} = E_v + E_p$$

Elasticity: stress & strain

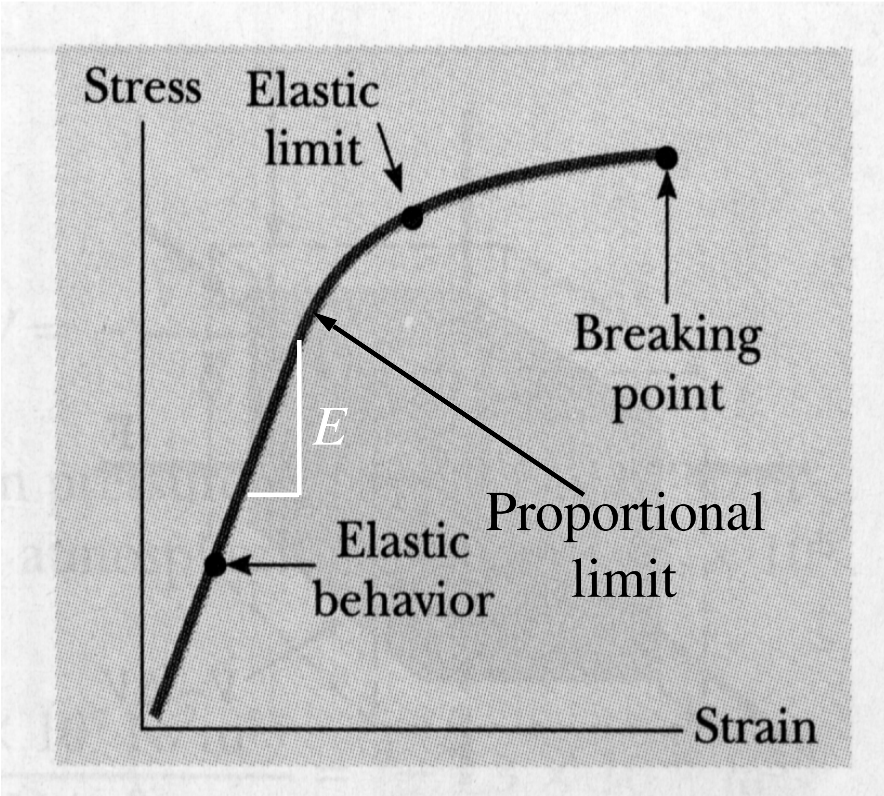


Stress: $\sigma = \frac{F}{A} = \frac{force}{surface}$

Stretch: $\lambda = \frac{l}{l_o}$

Strain: $\varepsilon = \frac{l - l_o}{l_o} = \lambda - 1$

Elastic Modulus

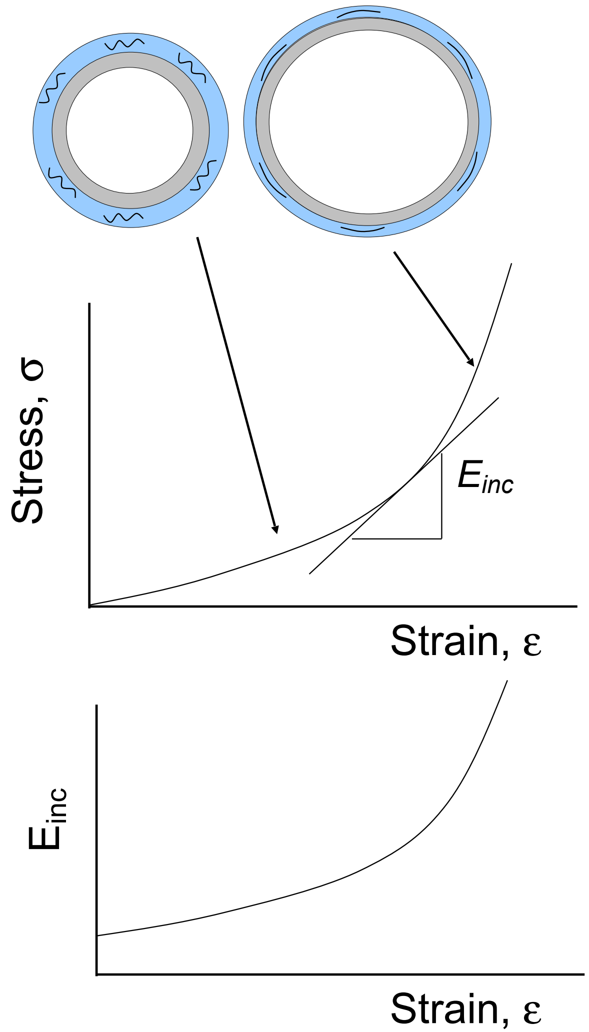


Hooke's law : $\sigma = E \cdot \varepsilon$

E: elastic modulus
or Young's elastic modulus

Material obeying Hooke's law:
Hookean or linear elastic

Incremental elastic modulus (E_{inc})

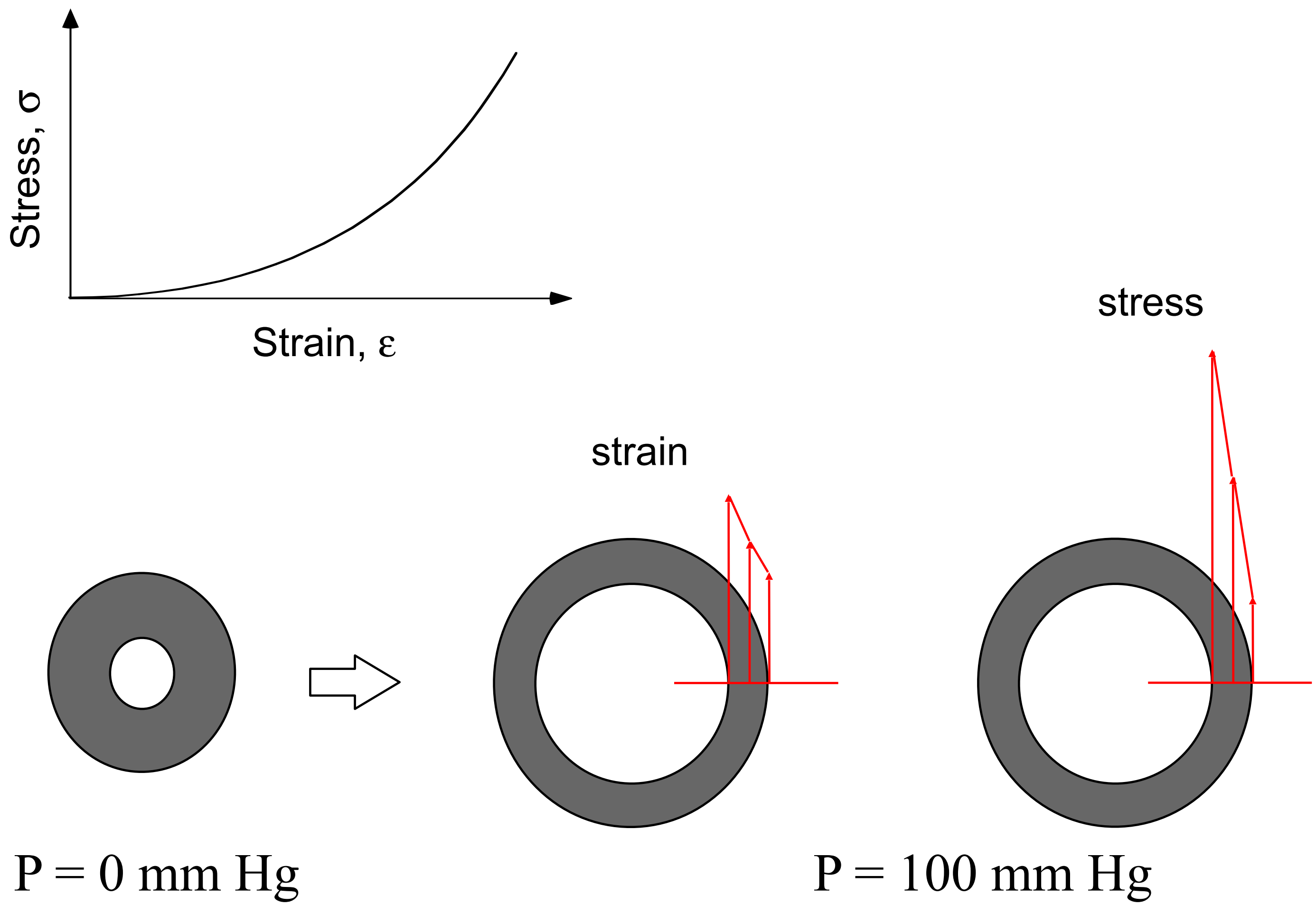


Nonlinear empirical relation between stress and strain

$$\sigma = a\left[e^{b(\lambda-1)} - 1\right]$$

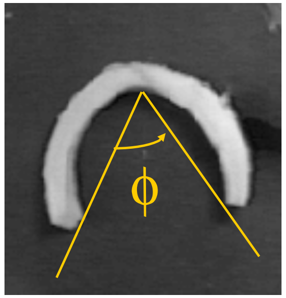
$$\sigma = a\left[e^{b\varepsilon} - 1\right]$$

Stress and strain in absence of prestress



Zero Stress State (ZSS), Opening angle, and Prestress

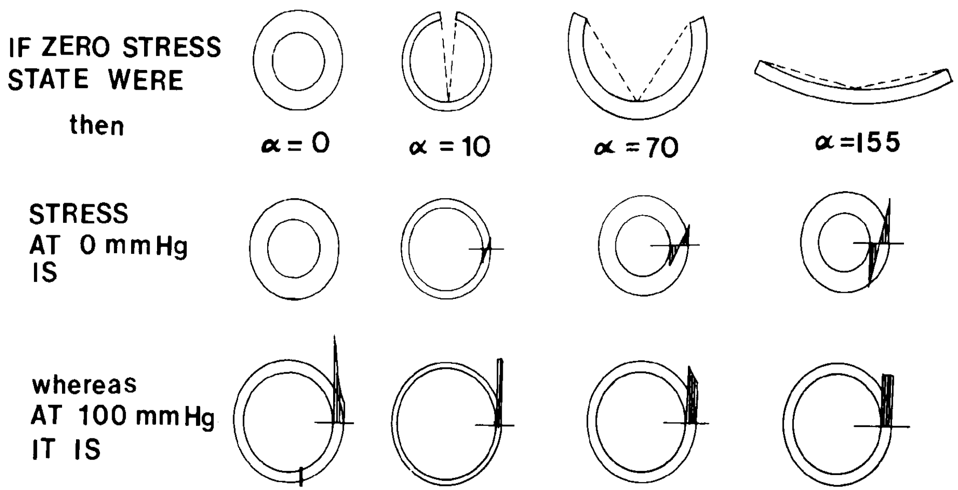
Arterial ring
+
longitudinal cut



Pig aorta

Zero Stress State:
opened-up configuration

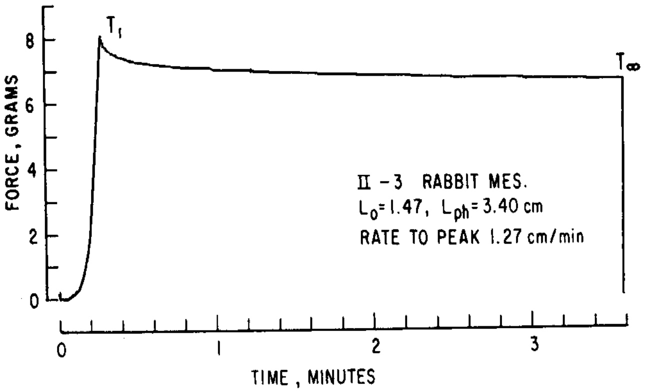
ϕ: opening angle



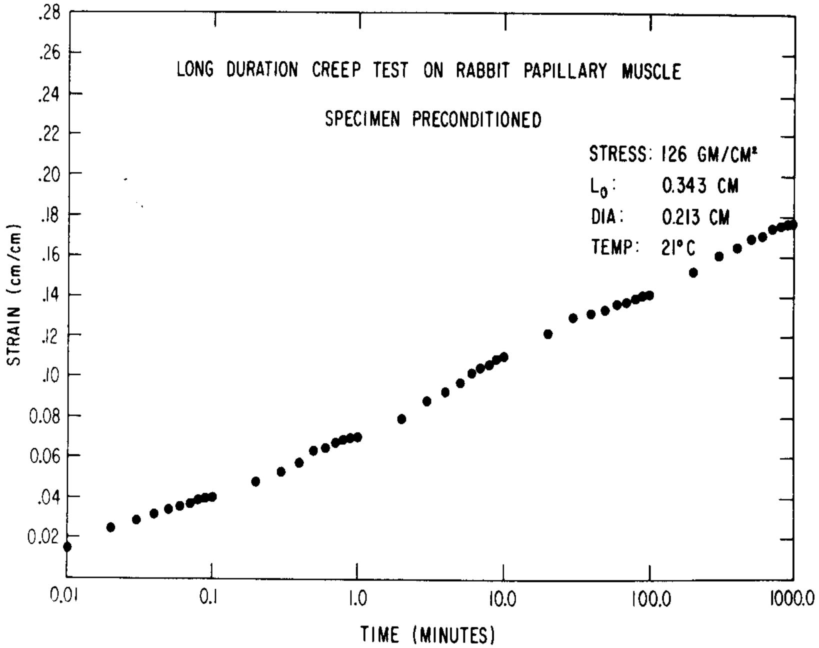
Prestress

Viscoelastic effects: relaxation & creep

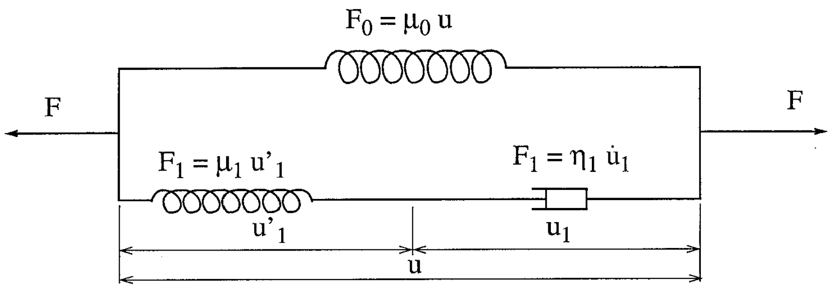
Relaxation



Creep



Standard viscoelastic model (Kelvin)



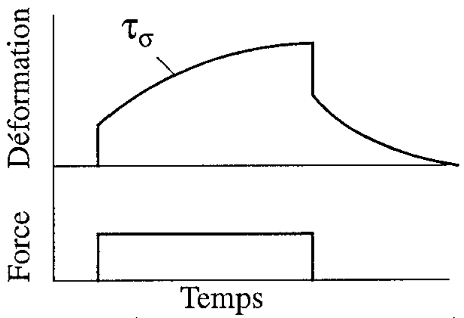
$$\left.\begin{aligned} F &= F_0 + F_1 \\ u &= u_1 + u'_1 \end{aligned}\right\} \Rightarrow$$

$$F + \tau_\varepsilon \dot{F} = E_R(u + \tau_\sigma \dot{u})$$

avec

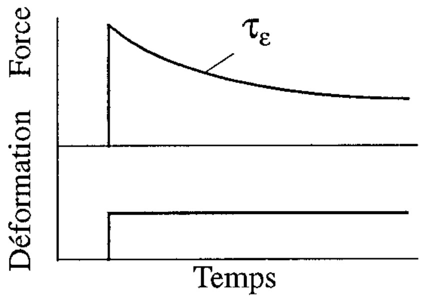
$$\tau_\varepsilon = \frac{\eta_1}{\mu_1} \qquad \tau_\sigma = \frac{\eta_1}{\mu_o}\left(1 + \frac{\mu_o}{\mu_1}\right)$$

Creep function



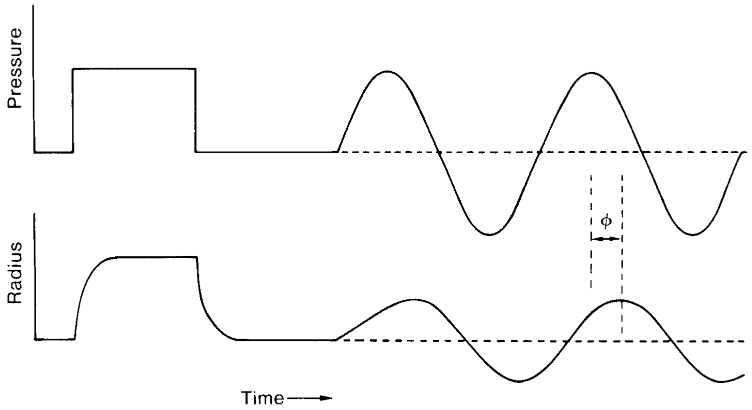
$$c(t) = \frac{1}{\mu_0}\left(1 - \left(1 - \frac{\tau_\varepsilon}{\tau_\sigma}\right) e^{-\frac{t}{\tau_\sigma}}\right)$$

Relaxation function



$$k(t) = \mu_0\left(1 - \left(1 - \frac{\tau_\sigma}{\tau_\varepsilon}\right) e^{-\frac{t}{\tau_\varepsilon}}\right)$$

Hysteresis

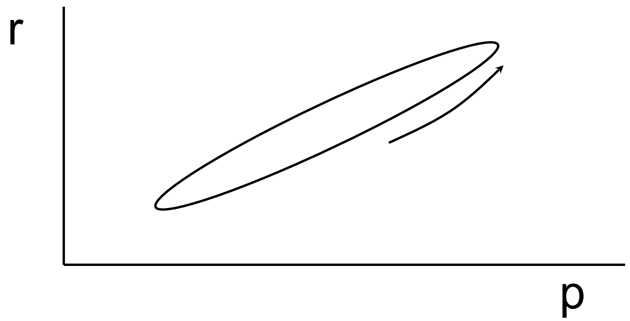


$$F = G(i\omega)u$$

$$G(i\omega) = |G|e^{i\delta}$$

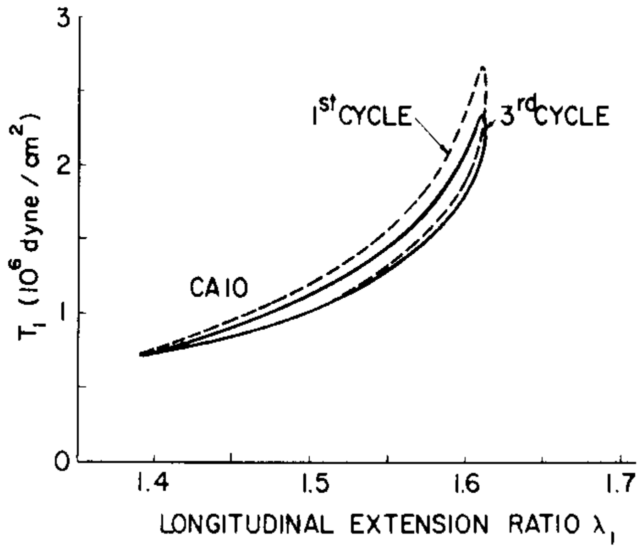
$$|G| = \mu_0\sqrt{\frac{1+\omega^2\tau_\sigma^2}{1+\omega^2\tau_\varepsilon^2}}$$

$$\tan\delta = \frac{\omega(\tau_\sigma - \tau_\varepsilon)}{1+\omega^2(\tau_\sigma \cdot \tau_\varepsilon)}$$

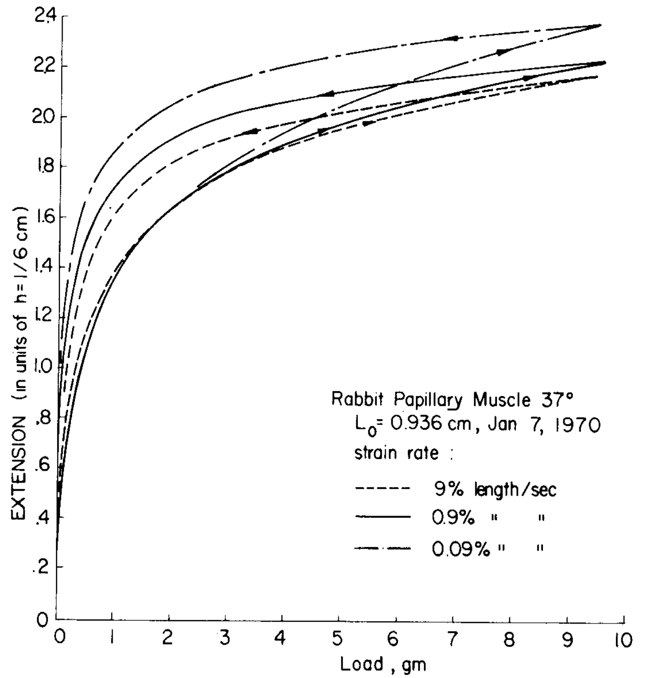


Preconditioning and hysteresis

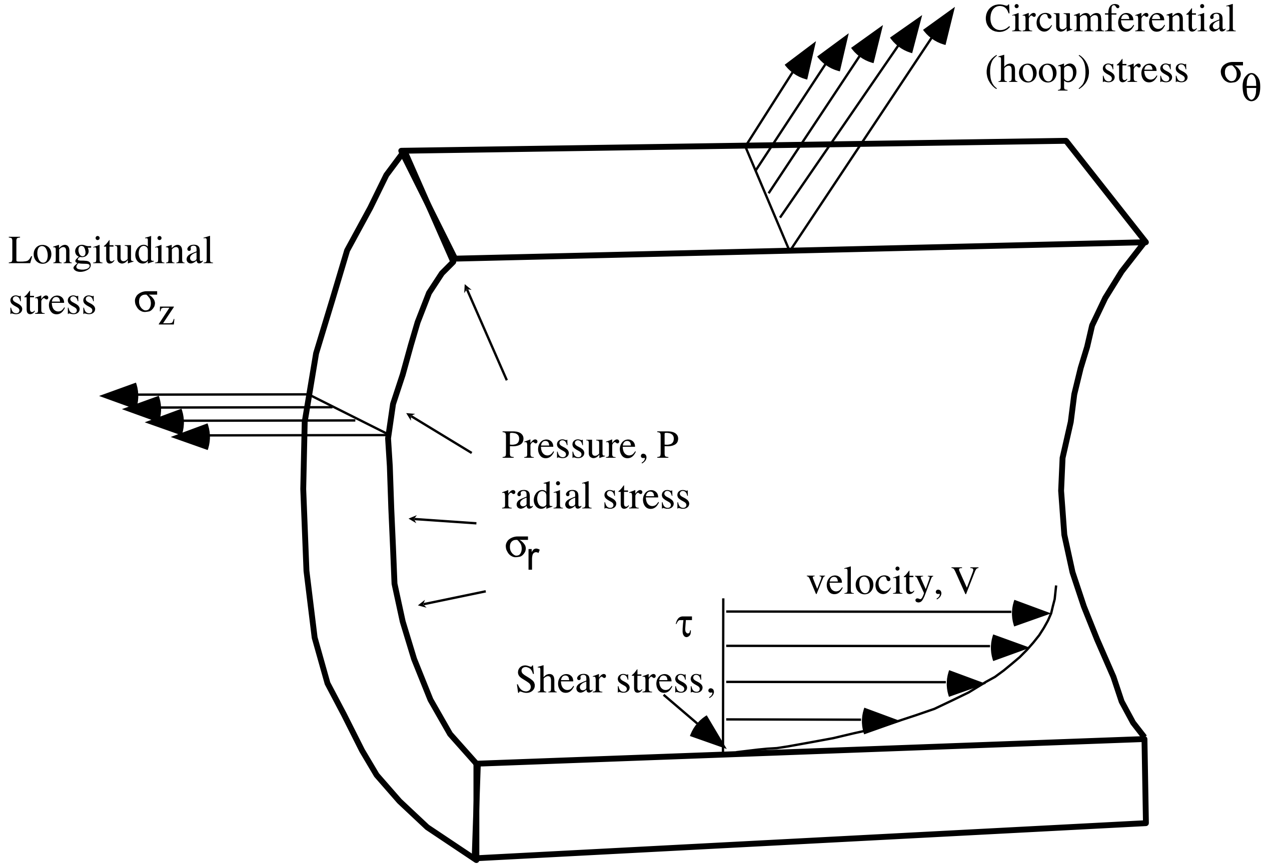
Preconditioning



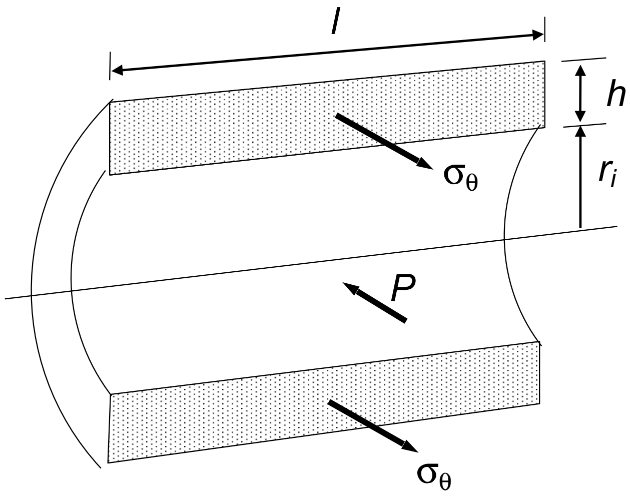
Hysteresis



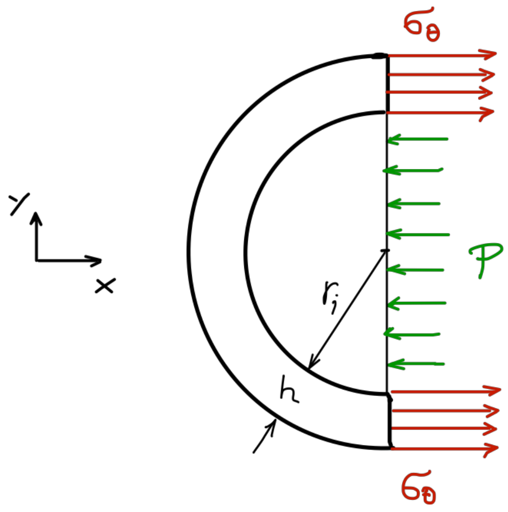
Stresses acting on the arterial wall



Law of Laplace (cylinder)



$$\sigma_\theta = \frac{P_i \cdot r_i}{h}$$

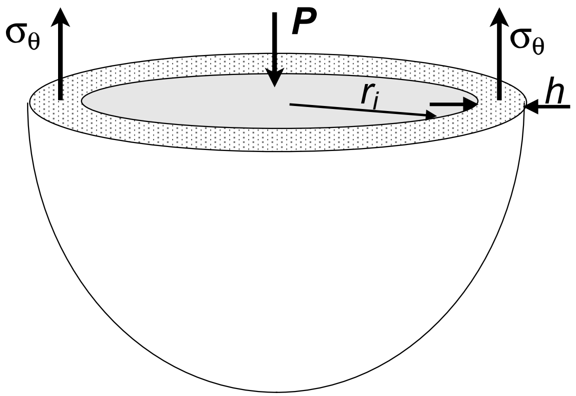


For equilibrium: $\sum F_x = 0$

$$2 \cdot \sigma_\theta \cdot h \cdot l = P \cdot (2 r_i \cdot l)$$

$$\Rightarrow \underline{\underline{\sigma_\theta = \frac{P \cdot r_i}{h}}}$$ Laplace's law

Law of Laplace (sphere)



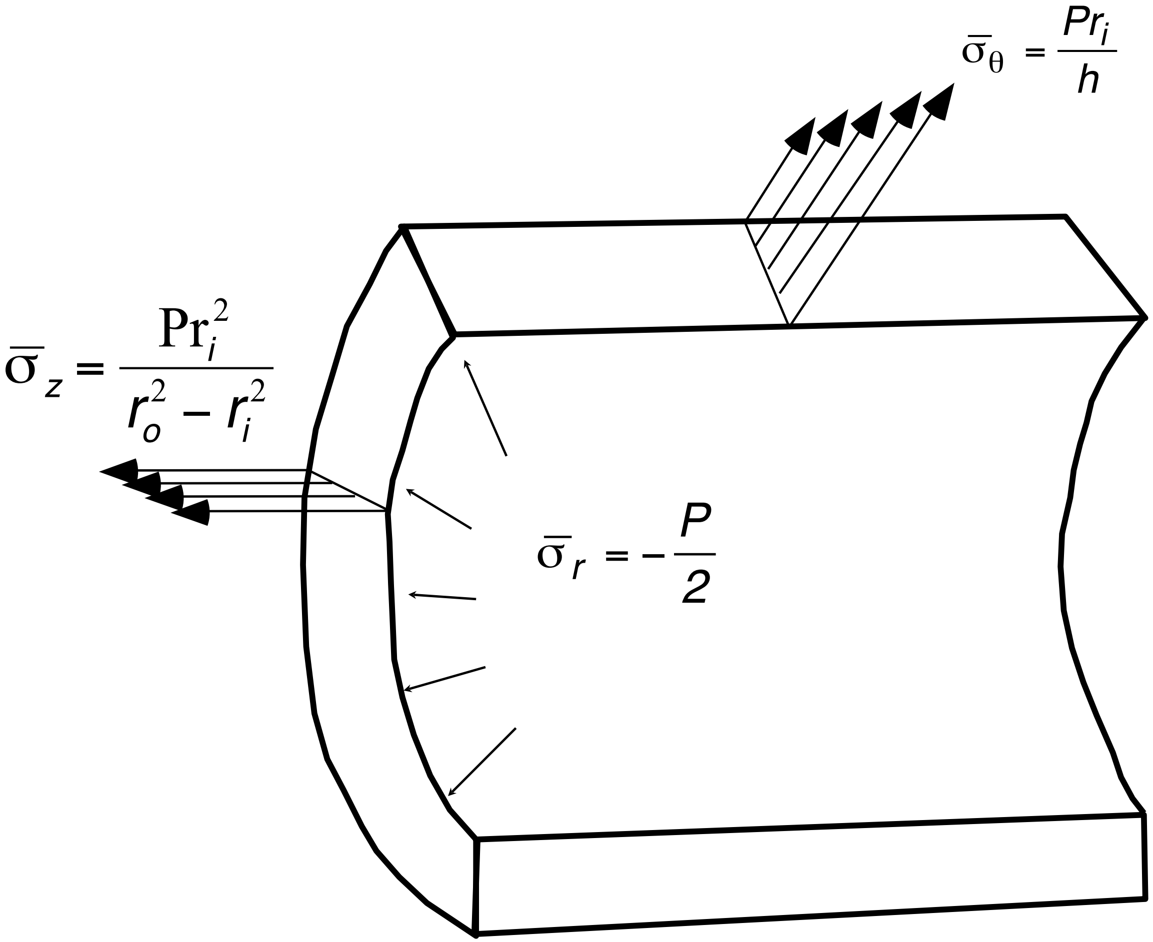
$$\sigma_\theta = \frac{P_i \cdot r_i}{2h}$$

$$\sum F_z = 0 \Rightarrow P \cdot \pi r_i^2 = \sigma_\theta \cdot \pi \left[(r_i + h)^2 - r_i^2 \right]$$

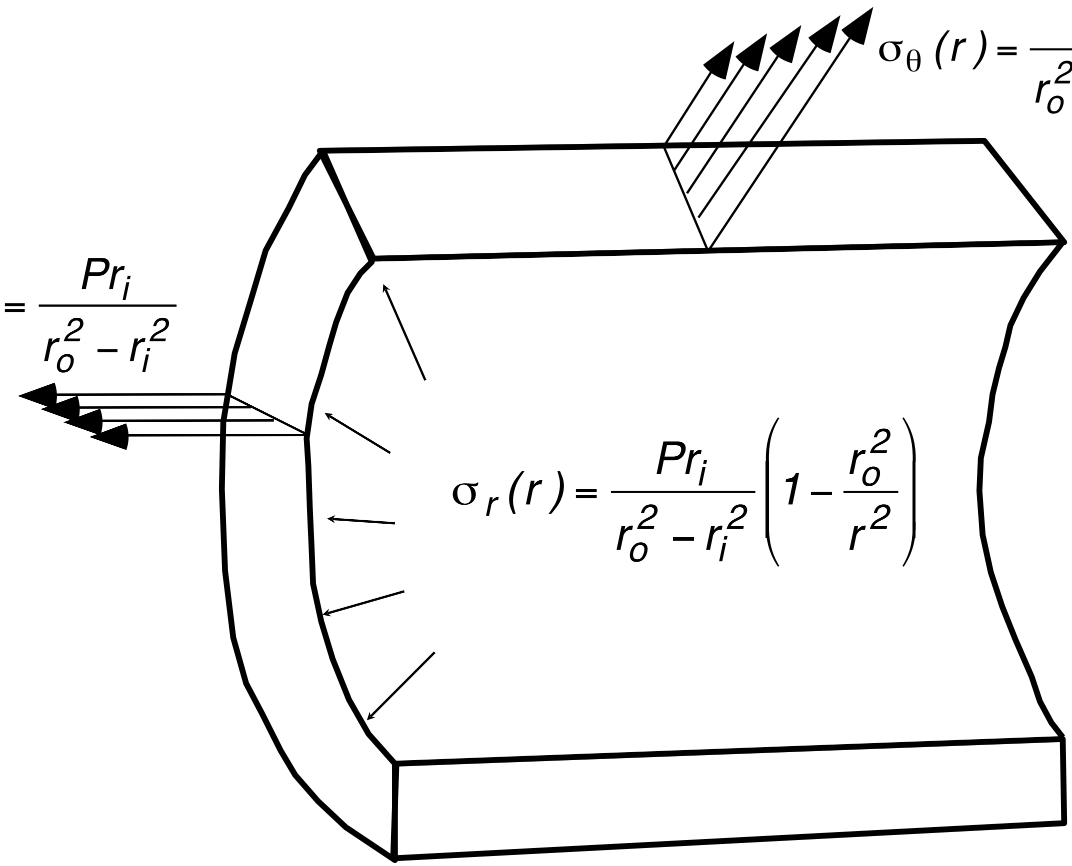
$$\Rightarrow \sigma_\theta = \frac{P \cdot r_i^2}{2 r_i h + h^2}$$

For $h \ll 2r_i$: $$\sigma_\theta = \frac{P \cdot r_i}{2h}$$

Average stresses



Wall Stress distribution (linear, small deformation)



$$\sigma_\theta(r) = \frac{Pr_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

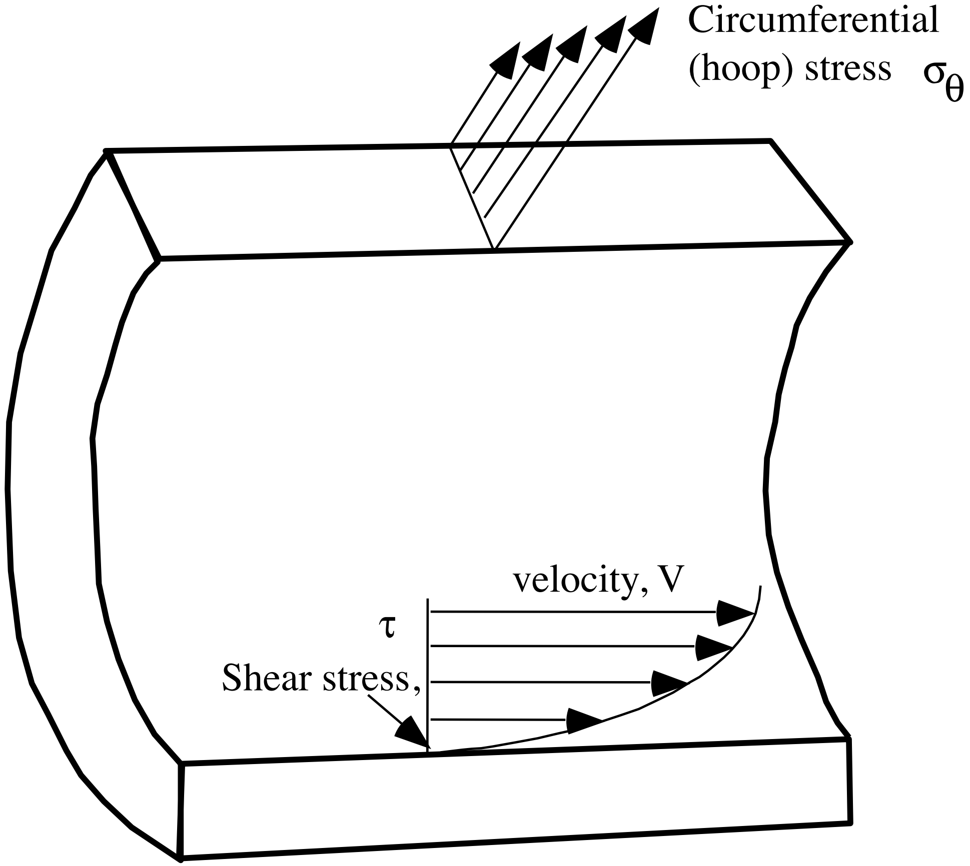
$$\sigma_z = \frac{Pr_i}{r_o^2 - r_i^2}$$

$$\sigma_r(r) = \frac{Pr_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right)$$

Hypotheses:
- small deformations
- Hookean material
- incompressible wall

<u>Not accurate</u> because of large deformation, nonlinearities
=> do not use!

Example: magnitude of hoop and shear stress acting on the aortic wall



Laplace:

$$\sigma_\theta = \frac{P \cdot r_i}{h} = \frac{100\ mmHg \cdot 1.3\ cm}{0.25\ cm} = 520\ mmHg \cdot \frac{133\ Pa}{1\ mmHg}$$

$$\Rightarrow \quad \sigma_\theta = 69'160\ Pa$$

Poiseuille:

$$\tau = \frac{4 \mu Q}{\pi r_i^3} = \frac{4 \times 3.5 \times 10^{-3}\ N \cdot s/m^2 \times 5 \times 10^{-3}\ m^3/60\,s}{\pi \cdot (1.3 \times 10^{-2})^3\ m^3}$$

$$\Rightarrow \quad \tau = 0.17\ Pa$$

Relation between E_{inc} et p(r)

$$E_{inc} = \frac{d\sigma}{d\varepsilon} \qquad \text{with} \quad d\varepsilon = \frac{2\pi\, dr}{2\pi r} = \frac{dr}{r}$$

$$\sigma = \frac{Pr}{h} \Rightarrow d\sigma = \frac{dp.r}{h} + \frac{p\,dr}{h} - \frac{Pr}{h^2}dh \quad (1)$$

Incompressibility condition (thin wall): $r.h = ct$

$$\Rightarrow \quad dr.h + r\,dh = 0$$

$$\Rightarrow \quad dr.h = -r\,dh \quad (2)$$

$$(1) \wedge (2) \Rightarrow d\sigma = \frac{dp.r}{h} + \frac{P\,dr}{h} + \frac{p\,dr\,h}{h^2}$$

$$\Rightarrow d\sigma = \frac{dp.r}{h} + 2\,\frac{P.dr}{h} \qquad (3)$$

Divide eq (3) by $d\varepsilon = \frac{dr}{r}$ to obtain:

$$\frac{d\sigma}{d\varepsilon} = \frac{dp.r}{h} \cdot \frac{r}{dr} + 2\,\frac{P.dr}{h} \cdot \frac{r}{dr}$$

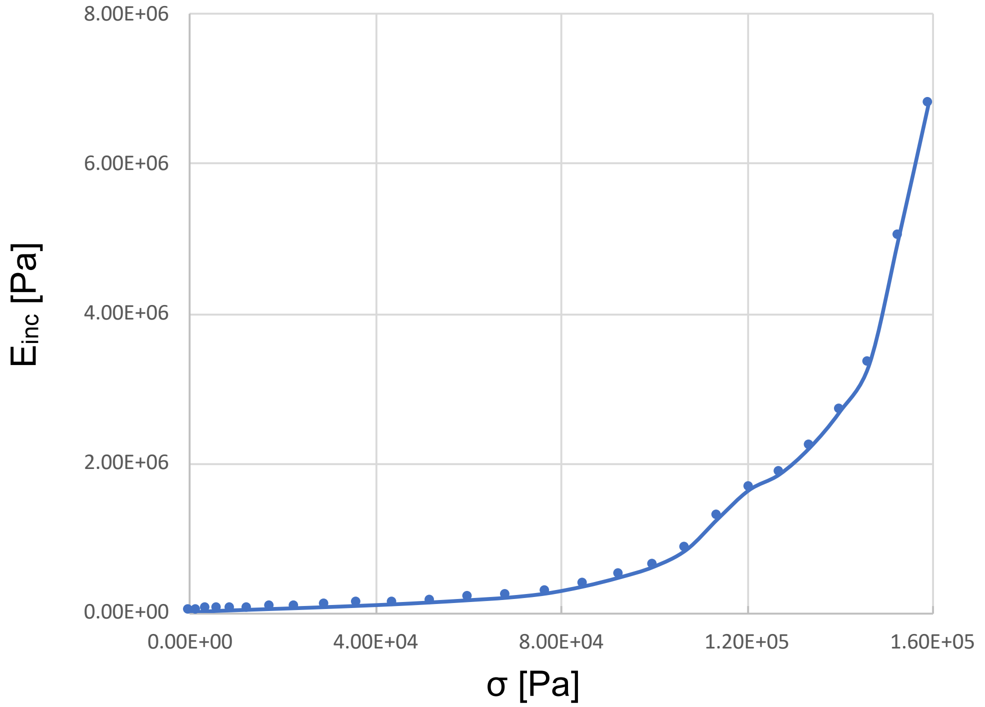
$$\Rightarrow \underbrace{\frac{d\sigma}{d\varepsilon}}_{E_{inc}} = \frac{r^2}{h}\,\frac{dp}{dr} + 2\,\underbrace{\frac{P.r}{h}}_{\sigma}$$

$$\Rightarrow \quad E_{inc} = \frac{r^2}{h}\,\frac{dp}{dr} + 2\sigma$$

In general $\sigma \ll E_{inc}$

$$\rightsquigarrow \quad E_{inc} \simeq \frac{r^2}{h}\,\frac{dp}{dr}$$

Relation between E_{inc} et p(r)

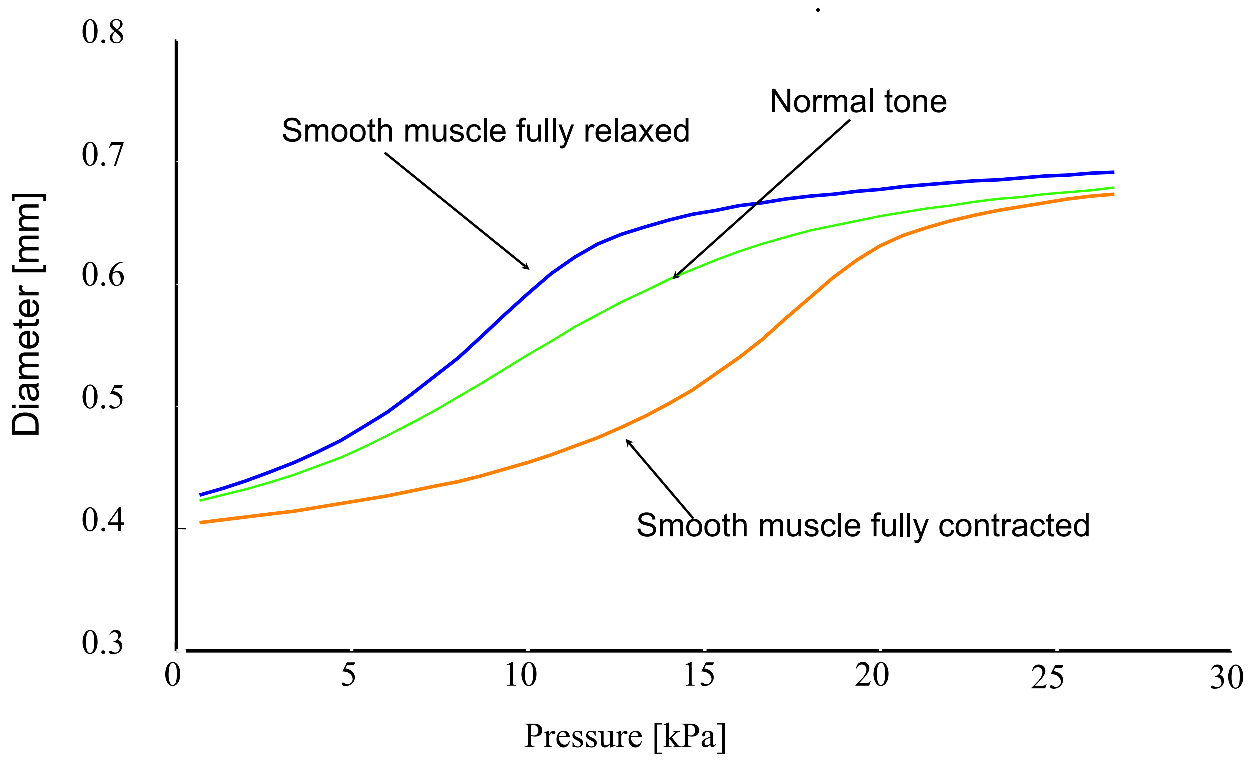


$E_{inc} >> \sigma$

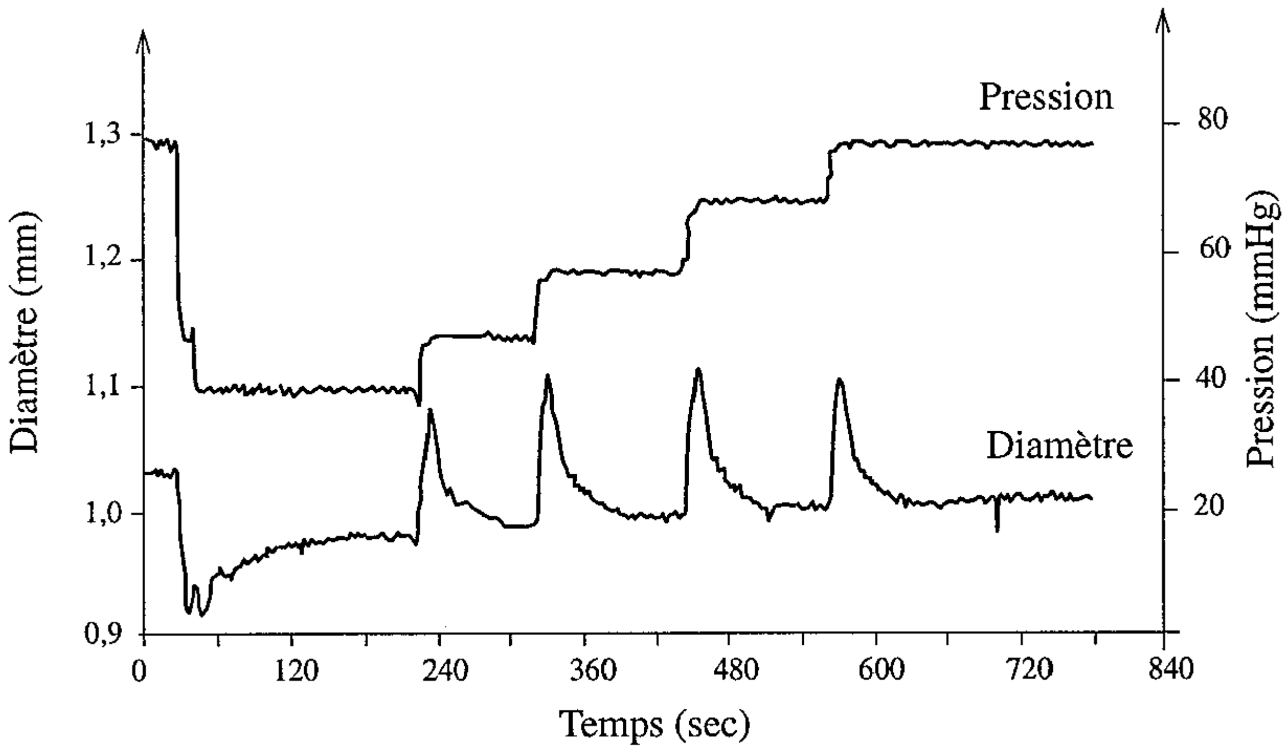
For a thin ($h << r_i$) and incompressible wall: $$E_{inc} = \frac{r_i^2}{h}\frac{dP}{dr_i}$$

If the wall cannot be considered thin: $$E_{inc} = \frac{3}{2}\frac{r_i^2 r_o}{(r_o^2 - r_i^2)}\frac{\Delta P}{\Delta r_o}$$

Pressure-diameter relationship as a function of smooth muscle tone

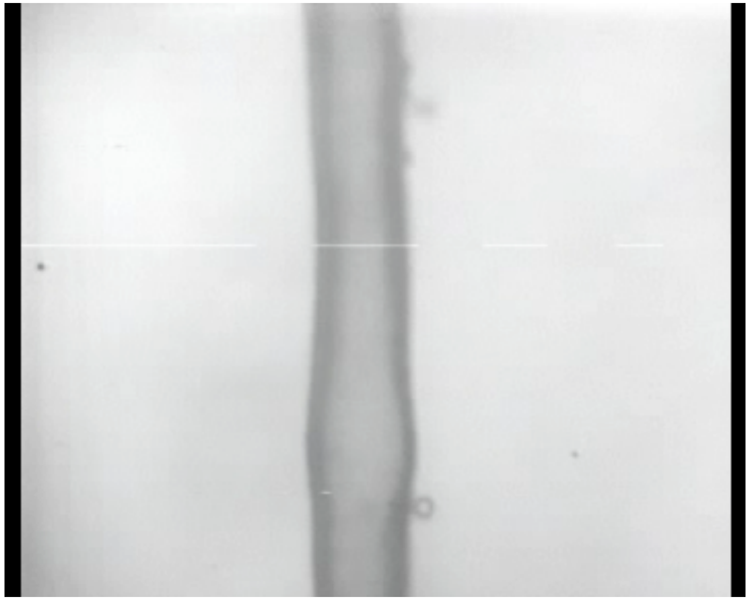


Myogenic tone



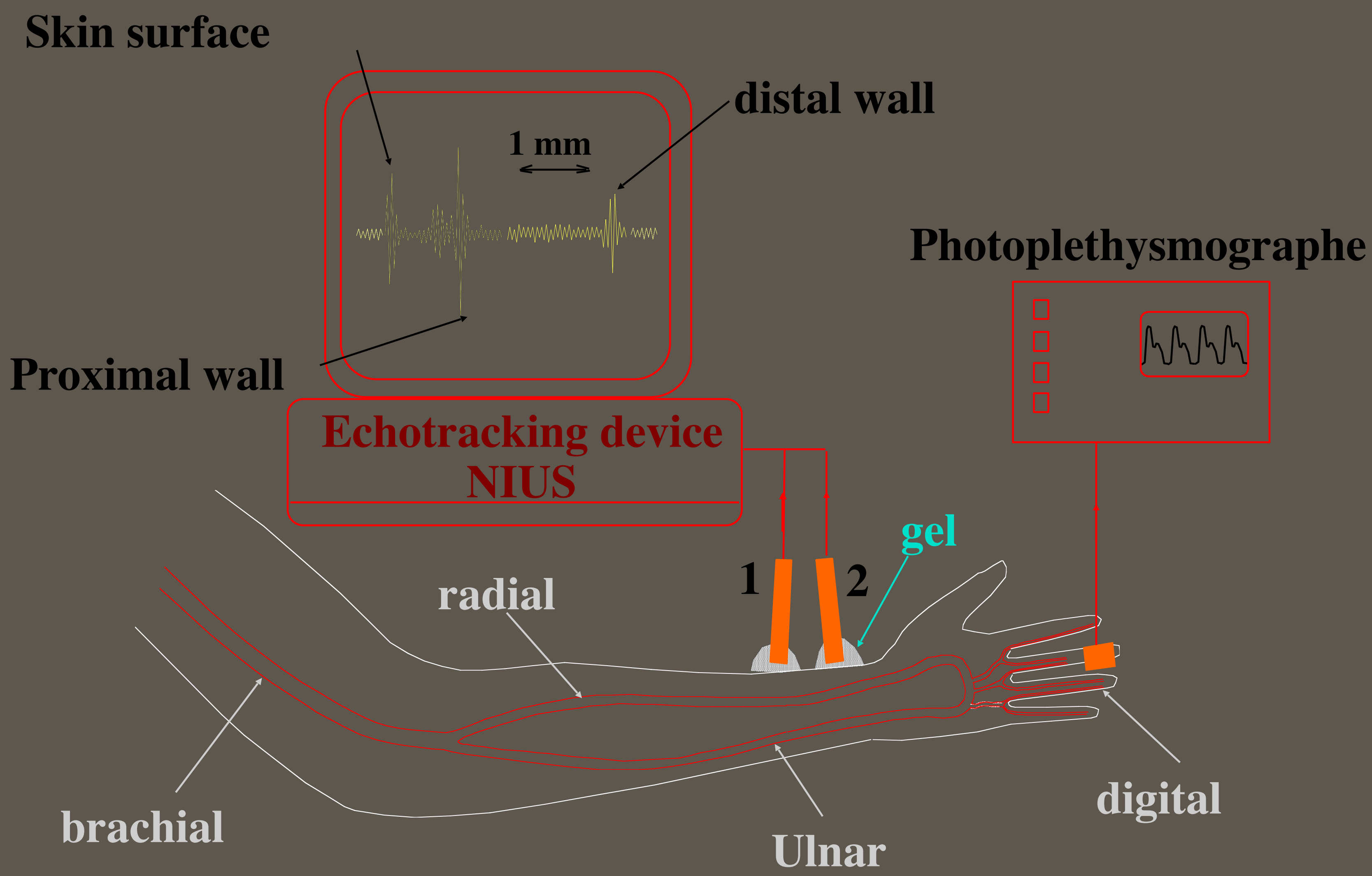
Changes in intraluminal pressure provoke an instantaneous lumen diameter extension (passive response) followed by a smooth muscle contraction, which tends to bring diameter back to the original level. This phenomenon is termed **myogenic response**

Vasomotion

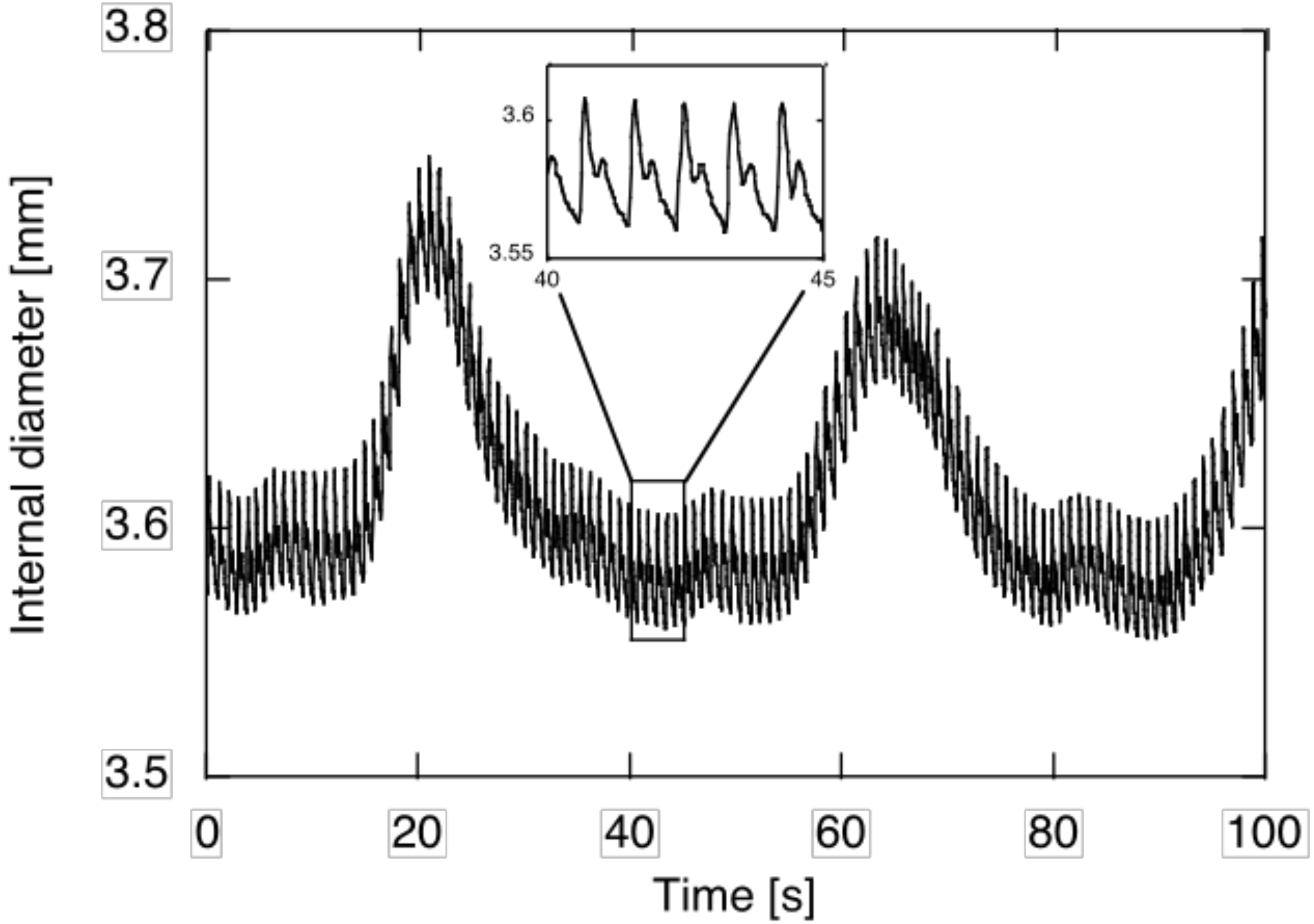


Spontaneous oscillations in diameter of an in vitro perfused rat carotid artery (**vasomotion)**

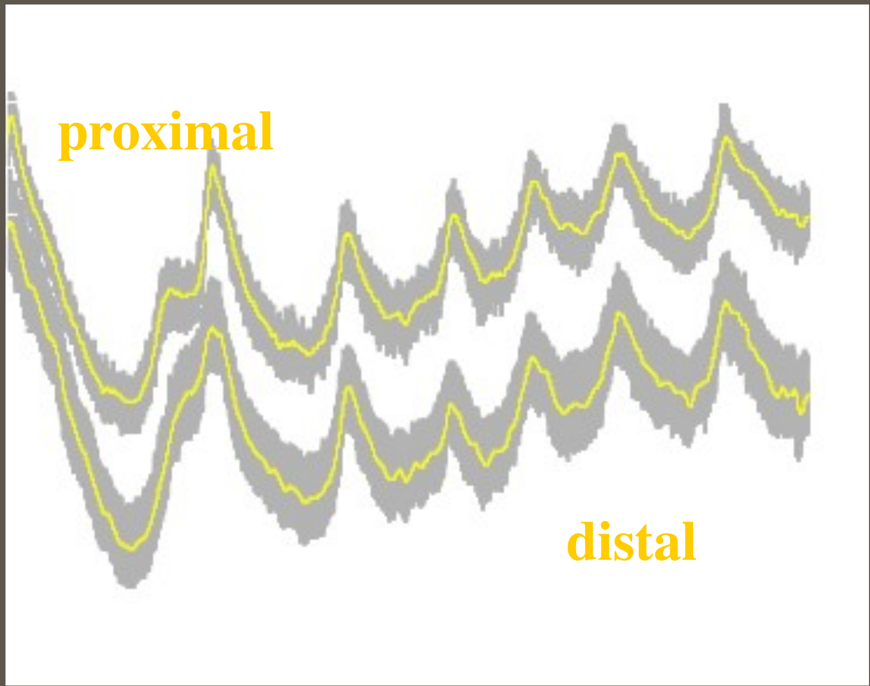
Experimental setup for vasomotion studies



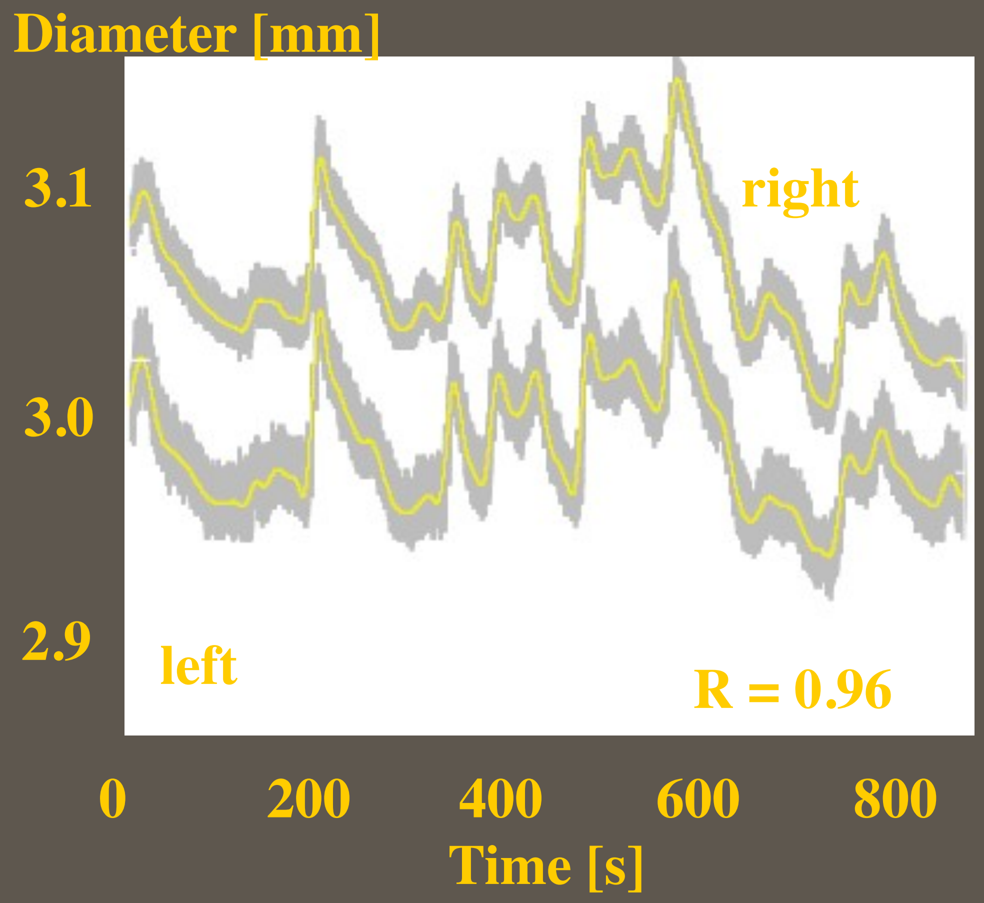
Vasomotion (human radial artery)



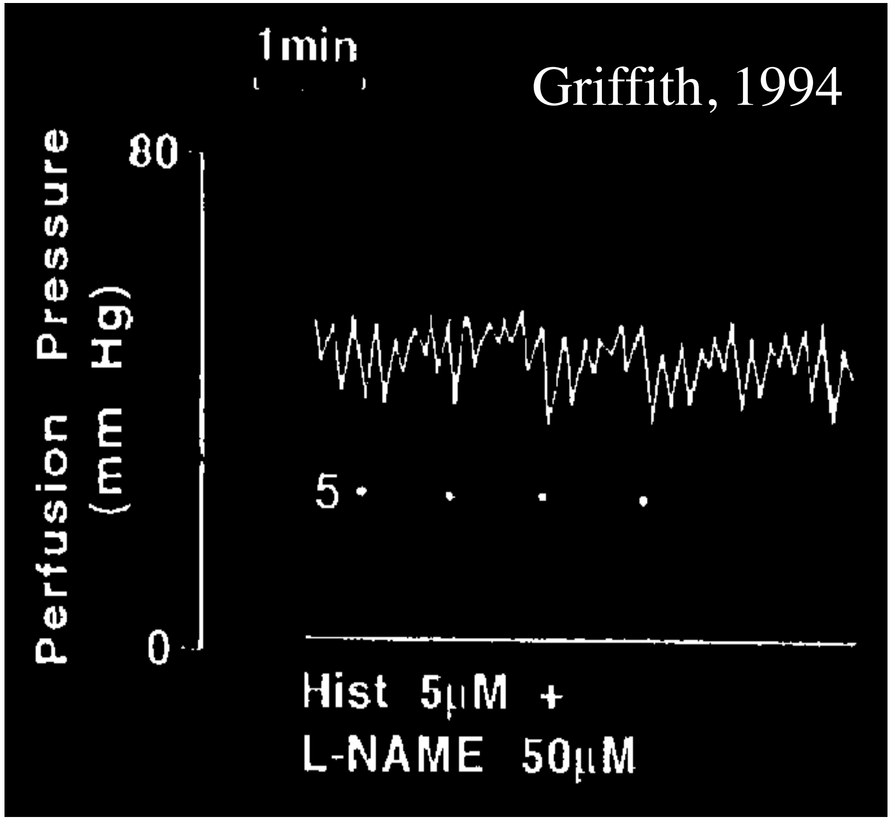
Simultaneous ipsilateral measurements on the radial artery

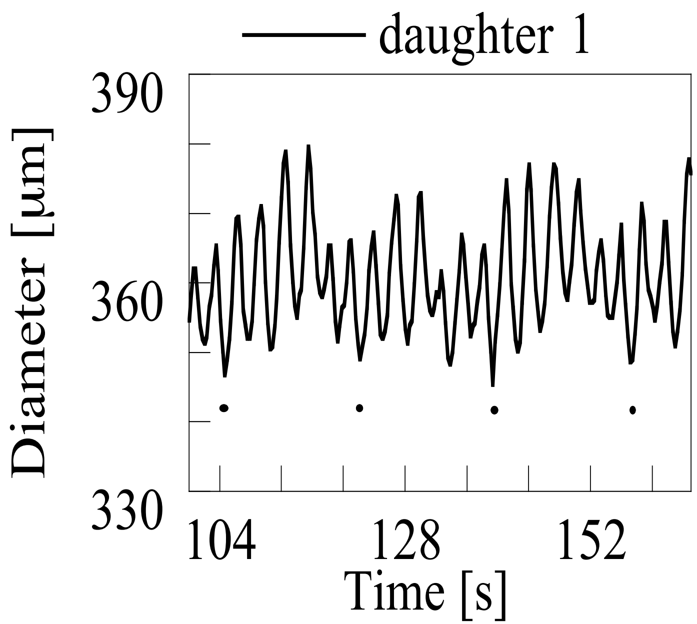


Simultaneous contralateral radial artery diameter measurements



Higher order periods





Routes to chaos: intermittencies